COMPOSITE MATERIALS DESIGN

4 Micromechanical analysis of composite materials

Outline

- elastic deformation of anisotropic materials
- off-axis elastic constant of laminae
- elastic deformation of laminates
- stresses and distorsions

So far we have talked on apparent homogenized properties of a fiber reinforced lamina. Now we will examine how we can calculate the homogeneous lamina properties from the heterogeneous composite material constituent properties.

Micromechanics: Study of mechanical behavior of a composite material in terms of its constituent materials”

Macromechanics: Study of mechanical behavior of a homogenized composite material.

Constituent Materials:

- **Fiber** (Graphite, boron, Silicon): Ef , σf , Gf, and Vf
- **Matrix** (Resin): Em , σ m , Gm Vm

Volume fraction

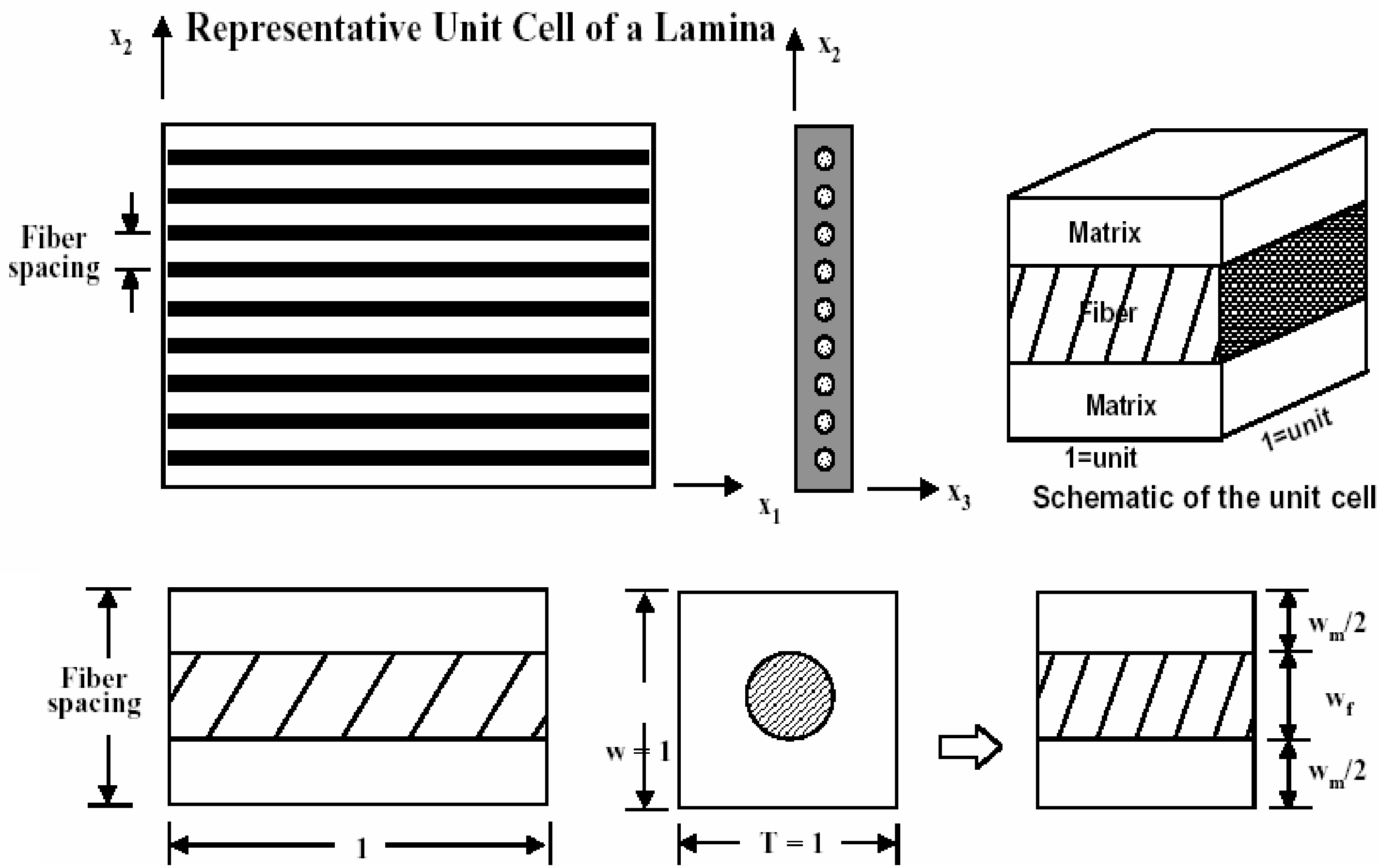
Vf = Fiber volume fraction = Vol. of fiber / Total volume
Vm = Matrix volume fraction = volume of matrix / Total volume

If no voids in the composites.
Then Vf + Vm = volume of composite = 1

Assumptions in Micromechanics of Composites

• The Lamina is :	**Macroscopically homogeneous** **Linearly elastic** **Macroscopically Orthotropic** **Initially stress free**
• The fibers are :	**Homogeneous** **Linearly elastic** **Isotropic/Orthotropic** **Regularly spaced** **Perfectly aligned**
•The matrix is :	**Homogeneous** **Linearly elastic** **Isotropic**

Mechanics of Materials Method



(Area) Volume of fiber = Af
(Area) Volume of matrix = Am
(Area) Volume of composite = Ac

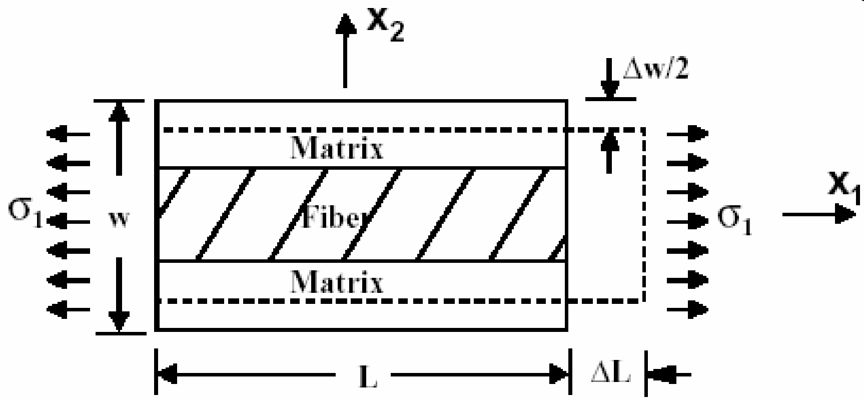
Fiber volume fraction = Af/Ac = Vf
Matrix volume fraction = Am/Ac=Vm

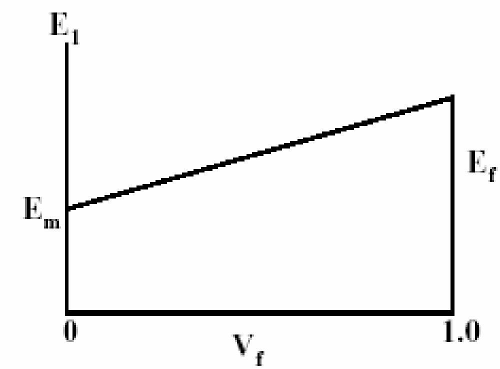
Analysis Comp

Determination of E_1 and σ_1

$E_{comp} = E_f V_f + E_m V_m$

(a) Determination of E_1





σ

Assumption: Axial (x-) strain is same for the lamina, fiber and matrix

Strain in the composite

$$\varepsilon_1 = \frac{\Delta L}{L} = \varepsilon_f = \varepsilon_m$$

Total force in composite

$$\sigma_1 A_c = \sigma_f A_f + \sigma_m A_m$$

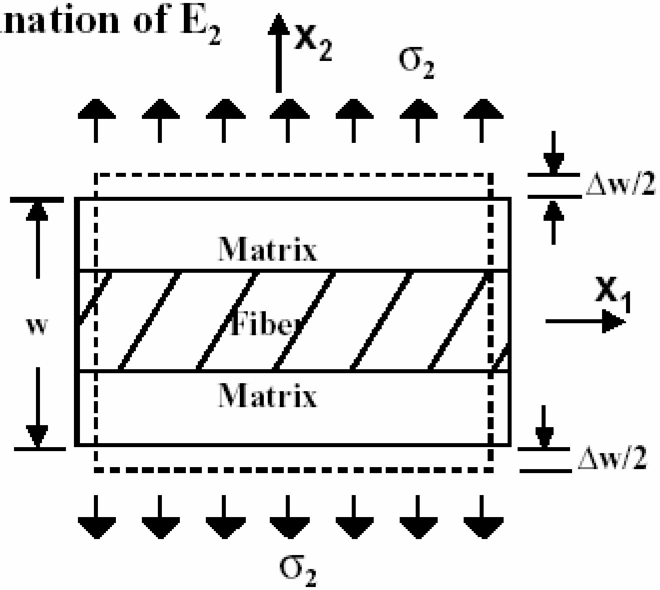
Stress in the composite

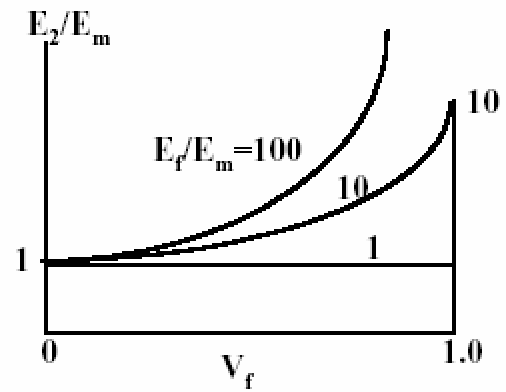
$$\boxed{\sigma_1 = \sigma_f \frac{A_f}{A_c} + \sigma_m \frac{A_m}{A_c} = \sigma_f V_f + \sigma_m V_m}$$

$$\varepsilon_1 E_1 = \varepsilon_1 E_f V_f + \varepsilon_1 E_m V_m$$

Determination of E_2 and σ_2

Determination of E_2





Assumption: Transverse stress, σ2, is same in composite, fiber, and the matrix

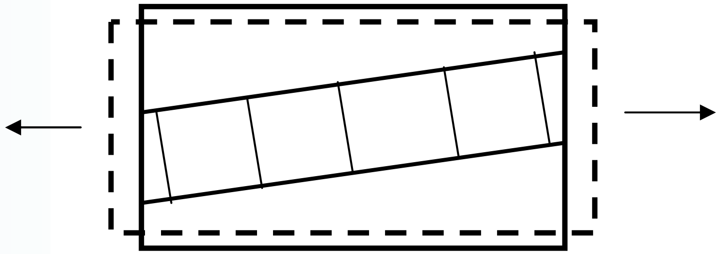
$$\Delta W = \varepsilon_2 W = \frac{\sigma_2}{E_f} W_f + \frac{\sigma_2}{E_m} W_m$$

$$or \quad \frac{\sigma_2}{E_2} = \sigma_2 \frac{W_f}{E_f W} + \sigma_2 \frac{W_m}{E_m W}$$

1/E = Vf/Ef + Vm/Em

E_2 = (Em·Ef)/ (EfVm+EmVf)

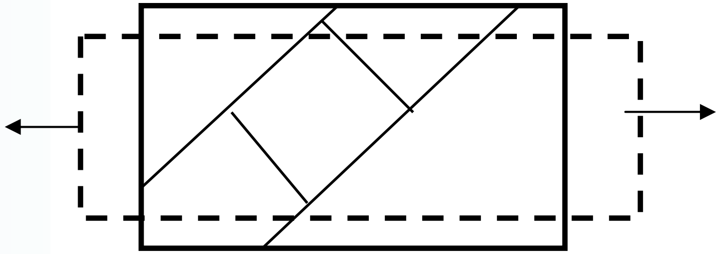
Determination of $E\alpha$ and $\sigma\alpha$



If $0< \alpha< 45$

$$E_{\alpha} = E_f V_f \cos^4 \alpha + E_m V_m$$

$$\sigma_{\alpha} = \sigma_f V_f \cos^4 \alpha + \sigma_m V_m$$



If $45< \alpha< 90$

$$E_{\alpha} = (E_m \cdot E_f) / (\sin^4 \alpha\, E_f V_m + E_m \cdot V_f)$$

$$\sigma_{\alpha} = (\sigma m \cdot \sigma f) / (\sin^4 \alpha\, \sigma_f V_m + \sigma_m \cdot V_f)$$

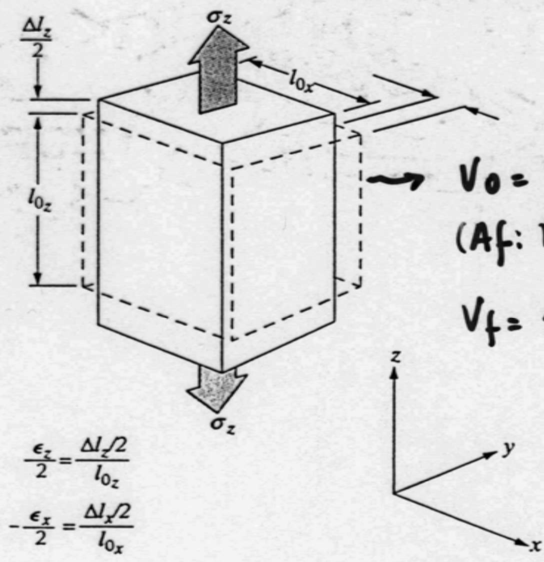
If $\alpha = 45^o, 135^o, 225^o$ and 315^o

$$E\,\alpha = [(E_m \cdot E_f) / (\sin^4 \alpha\, E_f V_m + E_m \cdot V_f) + E_f V_f \cos^4 \alpha + E_m V_m] / 2$$

$$\sigma\,\alpha = [(\sigma_m \cdot \sigma_f) / (\sin\,\alpha\, \sigma_f V_m + \sigma_m \cdot V_f) + \sigma_f V_f \cos^4 \alpha + \sigma_m V_m] / 2$$

Poisson's coeficient (ν)

$$\nu = \frac{\varepsilon_x}{\varepsilon_z} = \frac{\varepsilon_y}{\varepsilon_z}$$

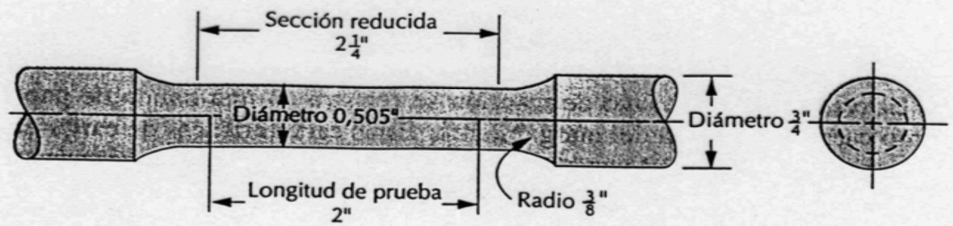


$V_0 = 1m^3$ (1x1x1)

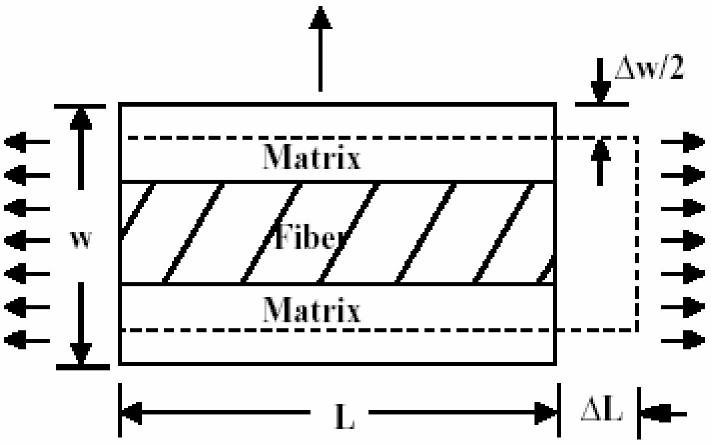
(Af: P_z = 1000 Pa)

$V_f = 1m^3$ (1,3x0.877x0.877)

$$\nu = \frac{0.123}{0.3} = 0.4$$



Determination of ν_{12}



Lateral strain due to σ_1

$$\varepsilon_2 = \frac{\Delta W}{W}$$

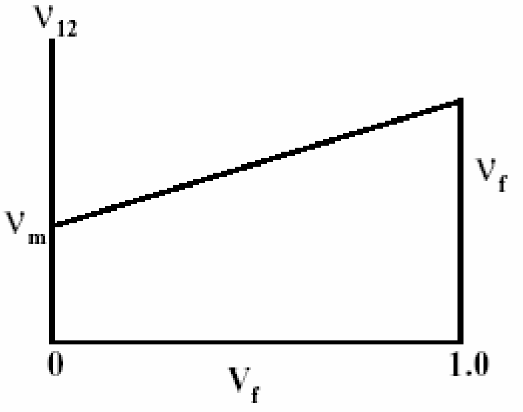
Poisson ratio

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$$

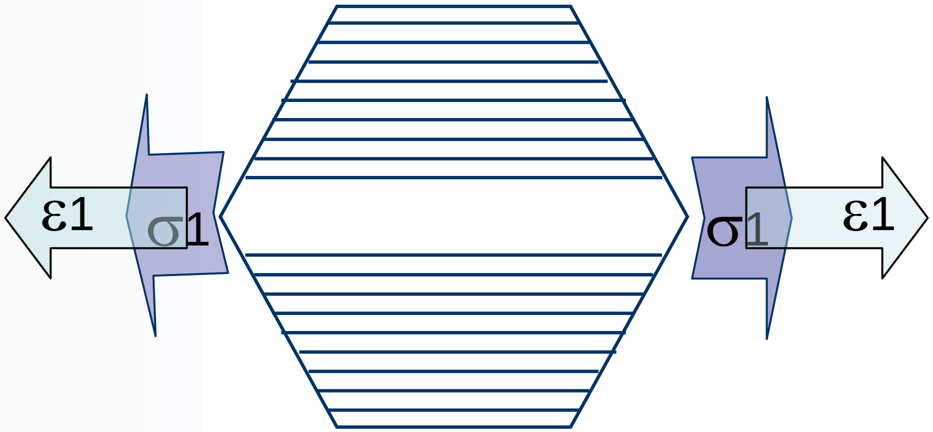
$$\varepsilon_2 = \frac{\Delta W}{W} = \frac{-\varepsilon_1 \nu_f W_f - \varepsilon_1 \nu_m W_m}{W}$$

$$\varepsilon_2 = -\varepsilon_1 \nu_f V_f - \varepsilon_1 \nu_m V_m$$

$$\boxed{\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1} = \nu_f V_f + \nu_m V_m}$$

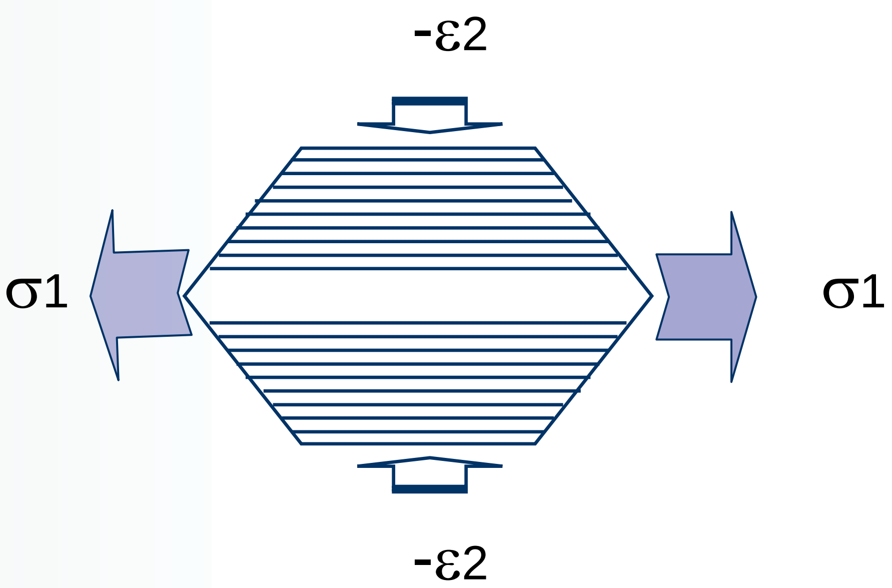


Deformation in x-way (ε_1) produced by y-load (σ_1)



$\varepsilon 1 = \sigma 1 / E1$

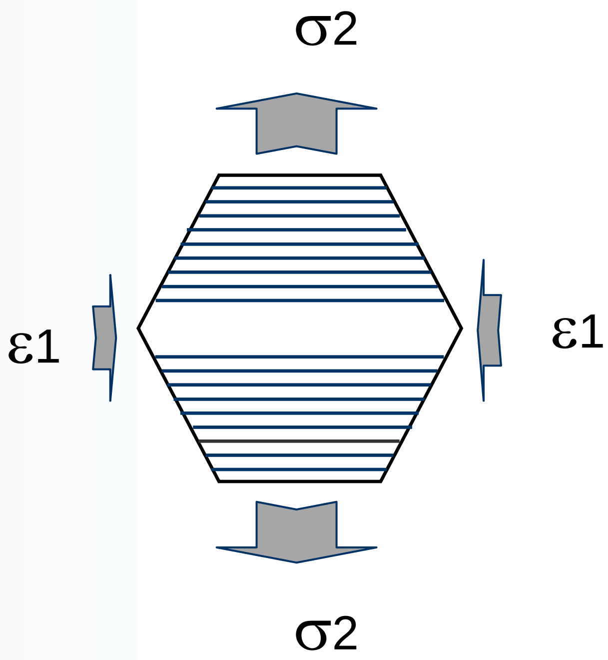
Contraction in y-way ($-\varepsilon 2$) produced by y-load ($\sigma 1$)



$$-\varepsilon 2 = (\nu 12 \cdot \sigma 1) / E1$$

$\nu 12$ $\begin{cases} \text{Load in 1} \\ \text{Contraction in 2} \end{cases}$

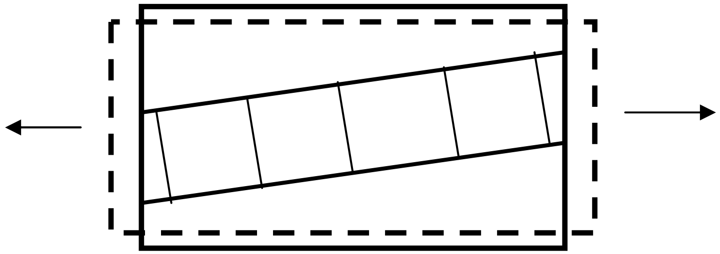
Contraction in x-way ($-\varepsilon_1$) produced by y-load (σ_2)



$$\varepsilon 1 = (\nu 21 \cdot \sigma 2) / E2$$

$$\nu 21 \begin{cases} \text{Load in 2} \\ \text{Contraction in 1} \end{cases}$$

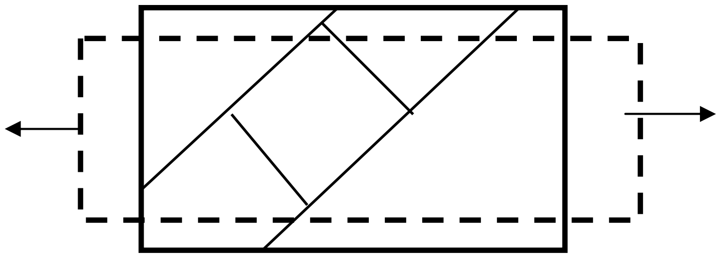
Determination of ν_α



If $0 < \alpha < 45$

$\nu_\alpha = \nu_f V_f \cos^4\alpha + \nu_m V_m$

(if $\alpha = 0$) $\nu_{12} = \nu_f V_f + \nu_m V_m$



If $45 < \alpha < 90$

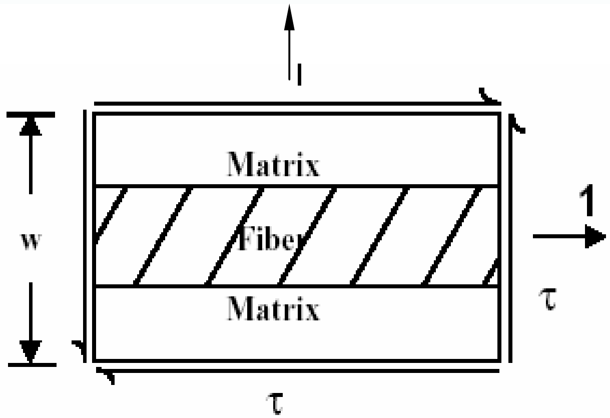
$\nu_\alpha = (\sin^4\alpha + \cos 4\alpha) \cdot (\nu_{12} \cdot E\alpha) / E_0$

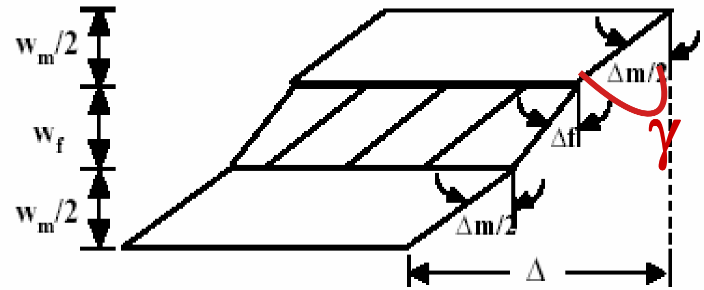
(if $\alpha = 90$) $\nu_\alpha = \nu_{21} = (\nu_{12} \cdot E_{90}) / E_0$

If $\alpha = 45^o, 135^o, 225^o$ and 315^o

$$\nu_\alpha = [(\sin^4\alpha + \cos^4\alpha) \cdot (\nu_{12} \cdot E_\alpha) / E_0 + (\nu_f V_f \cos^4\alpha + \nu_m V_m)]/2$$

Determination of G_{12}





Shear strain in Composite: $\gamma = \dfrac{\tau}{G_{12}}$

Shear strain in Matrix $\gamma_m = \dfrac{\tau}{G_m}$

Shear strain in Fiber $\gamma_f = \dfrac{\tau}{G_f}$

Total shear deformation $\Delta = \gamma W = \dfrac{\tau W}{G_{12}}$

Matrix $\Delta m = W_m \gamma_m$

Fiber $\Delta f = W_f \gamma_f$

$$\Delta = \frac{\tau W}{G_{12}} = \frac{W_m \tau}{G_m} + \frac{W_f \tau}{G_f}$$

$$\boxed{G_{12} = \frac{G_{12f} G_m}{V_f G_m + V_m G_{12f}}}$$

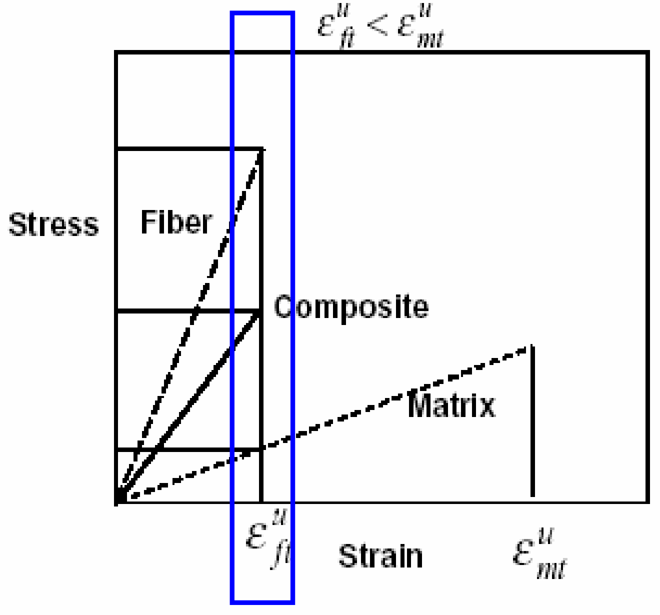
Halpin-Tsai's Equations

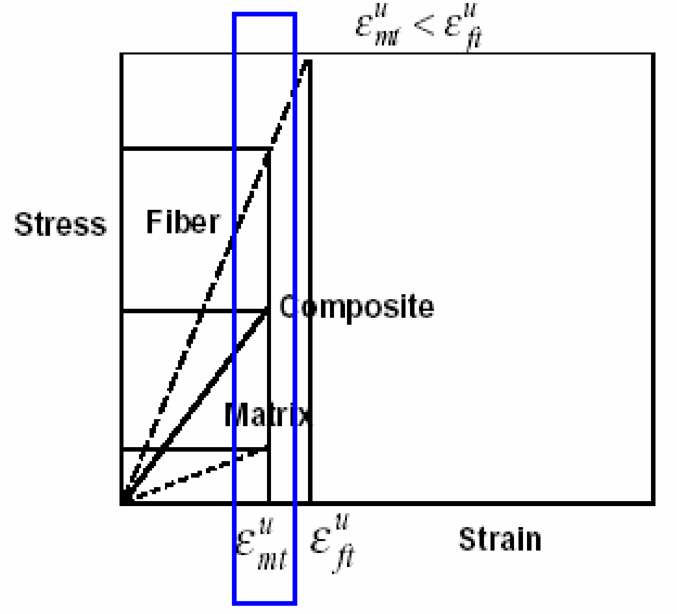
Of all the micromechanics equations Halpin- Tsai's semi-empirical equations are accurate and simple.
Halpin and Tsai showed that the Hermans solution to Hill's self consistent model can be reduced to the approximate form

$E_1 = E_f V_f + E_m V_m$	**Axial stiffness**
$\nu_{12} = \nu_f V_f + \nu_m V_m$	
$E_2 = E_m \dfrac{1+\xi_1 \eta_1 V_f}{1-\eta_1 V_f}$	**Transverse stiffness**
$\eta_1 = \dfrac{E_f - E_m}{E_f + \xi_1 E_m}$	**ξ Adjustable parameter**
$\boldsymbol{G}_{12} = \boldsymbol{G}_m \dfrac{1+\xi_2 \eta_2 \boldsymbol{V}_f}{1-\eta_2 \boldsymbol{V}_f}$	**Shear stiffness**
$\eta_2 = \dfrac{G_f - G_m}{G_f + \xi_2 G_m}$	

Analysis Comp

Longitudinal Tensile Strength (F1t)



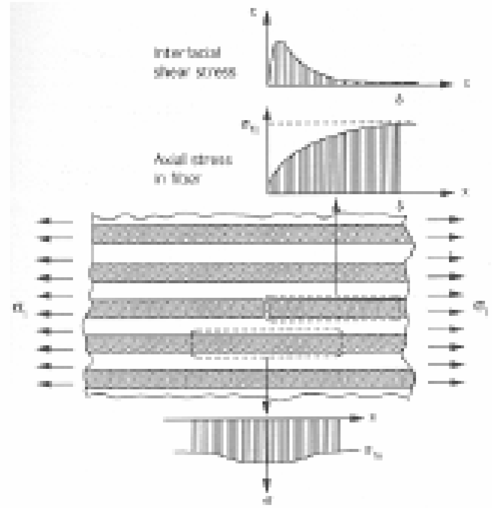


$$F_{1t} = E_f \varepsilon^u_{ft} V_f + E_m \varepsilon^u_{ft} V_m$$

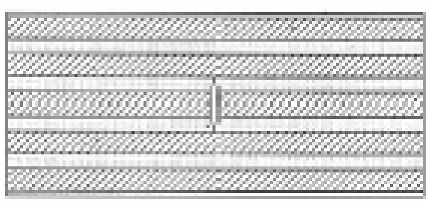
$$F_{1t} = E_f \varepsilon^u_{mt} V_f + E_m \varepsilon^u_{mt} V_m$$

Fiber Dominated Failure Mechanism

Fiber strength varies from point-point and fiber to fiber.

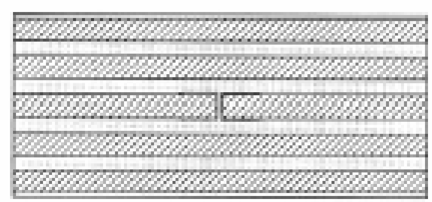


(a) Transverse Matrix Cracking



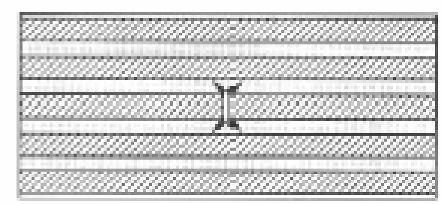
Brittle matrix & strong interface
Ceramic composites
(Silicon carbide/glass ceramic)

(b) Fiber Matrix Debond



Weak interface and/or high ultimate fiber strain
(Glass/epoxy)

(c) Conical Shear Fracture



Ductile matrix & strong interface
Metal matrix composites

Exercise 1

Computer the fiber volume fraction of an unidirectional layer with 5 tows per cm. Each tow is E-glass of 113 yield, and the final layer thickness is 2.0 mm.

Exercise 2

Computer the fiber volume fraction of a laminate 12.7 mm thick built with 22 layers of double bias (±45) at itched fabric with 5 tows per cm. Each tow is E-glass of 113 yield, and the final layer thickness is 2.0 mm.