

Digital Logic Gates

Logic gates are hardware blocks that produce signals equivalent to a binary 1 or 0 depending upon their input signals that are also equivalent to binary 1's or 0's.

Each gate has a distinct name and graphic symbol i.e.

AND gate:

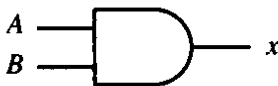

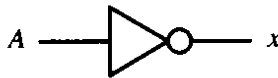


The operation of each gate can be described with an algebraic expression. i.e.






$$x = A \bullet B$$

Alternatively, the operation can be described using a truth table i.e.

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

Name	Graphic symbol	Algebraic function	Truth table															
AND		$x = A \cdot B$ or $x = AB$	<table><tr><th>A</th><th>B</th><th>x</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	x	0	0	0	0	1	0	1	0	0	1	1	1
A	B	x																
0	0	0																
0	1	0																
1	0	0																
1	1	1																
OR		$x = A + B$	<table><tr><th>A</th><th>B</th><th>x</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	x	0	0	0	0	1	1	1	0	1	1	1	1
A	B	x																
0	0	0																
0	1	1																
1	0	1																
1	1	1																
Inverter		$x = A'$	<table><tr><th>A</th><th>x</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>	A	x	0	1	1	0									
A	x																	
0	1																	
1	0																	

All digital functions can be implemented using these three gates.

Buffer	 $A \longrightarrow x \quad x = A$	<table><tr><th>A</th><th>x</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr></table>	A	x	0	0	1	1									
A	x																
0	0																
1	1																
NAND	 $A \quad B \longrightarrow x \quad x = (AB)'$	<table><tr><th>A</th><th>B</th><th>x</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	x	0	0	1	0	1	1	1	0	1	1	1	0
A	B	x															
0	0	1															
0	1	1															
1	0	1															
1	1	0															
NOR	 $A \quad B \longrightarrow x \quad x = (A + B)'$	<table><tr><th>A</th><th>B</th><th>x</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	x	0	0	1	0	1	0	1	0	0	1	1	0
A	B	x															
0	0	1															
0	1	0															
1	0	0															
1	1	0															
Exclusive-OR (XOR)	 $A \quad B \longrightarrow x$ $x = A \oplus B$ or $x = A'B + AB'$	<table><tr><th>A</th><th>B</th><th>x</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	x	0	0	0	0	1	1	1	0	1	1	1	0
A	B	x															
0	0	0															
0	1	1															
1	0	1															
1	1	0															
Exclusive-NOR or equivalence	 $A \quad B \longrightarrow x$ $x = (A \oplus B)'$ or $x = A'B' + AB$	<table><tr><th>A</th><th>B</th><th>x</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	x	0	0	1	0	1	0	1	0	0	1	1	1
A	B	x															
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Axioms and Theorems of Boolean Algebra

(1) Identity for +

$$\mathbf{x + 0 = x}$$

(2) Identity for •

$$\mathbf{x \cdot 1 = x}$$

(3) Dominator for +

$$\mathbf{x + 1 = 1}$$

(4) Dominator for •

$$\mathbf{x \cdot 0 = 0}$$

(5) Idempotence

$$\mathbf{x + x = x}$$

(6) Idempotence

$$\mathbf{x \cdot x = x}$$

(7) Complements

$$\mathbf{x + x' = 1}$$

(8) Complements

$$\mathbf{x \cdot x' = 0}$$

(9) Commutative

$$\mathbf{x + y = y + x}$$

(10) Commutative

$$\mathbf{x \cdot y = y \cdot x}$$

(11) Associative

$$\mathbf{x + (y + z) = (x + y) + z}$$

(12) Associative

$$\mathbf{x \cdot (y \cdot z) = (x \cdot y) \cdot z}$$

(13) Distribution of • over +

$$\mathbf{x \cdot (y + z) = x \cdot y + x \cdot z}$$

(14) Distribution of + over •

$$\mathbf{x + (y \cdot z) = (x + y) \cdot (x + z)}$$

(15) DeMorgan's

$$\mathbf{(x + y)' = x' \cdot y'}$$

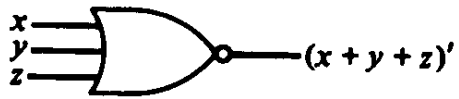
(16) DeMorgan's

$$\mathbf{(x \cdot y)' = x' + y'}$$

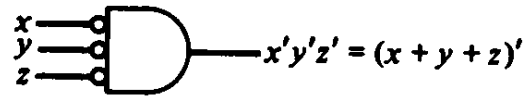
(17) Involution

$$\mathbf{(x')' = x}$$

NOR and NAND gates



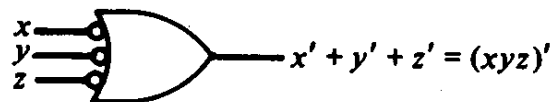
(a) OR-invert



(b) invert-AND



(a) AND-invert



(b) invert-OR

