BASIC CONCEPTS EXTENSION

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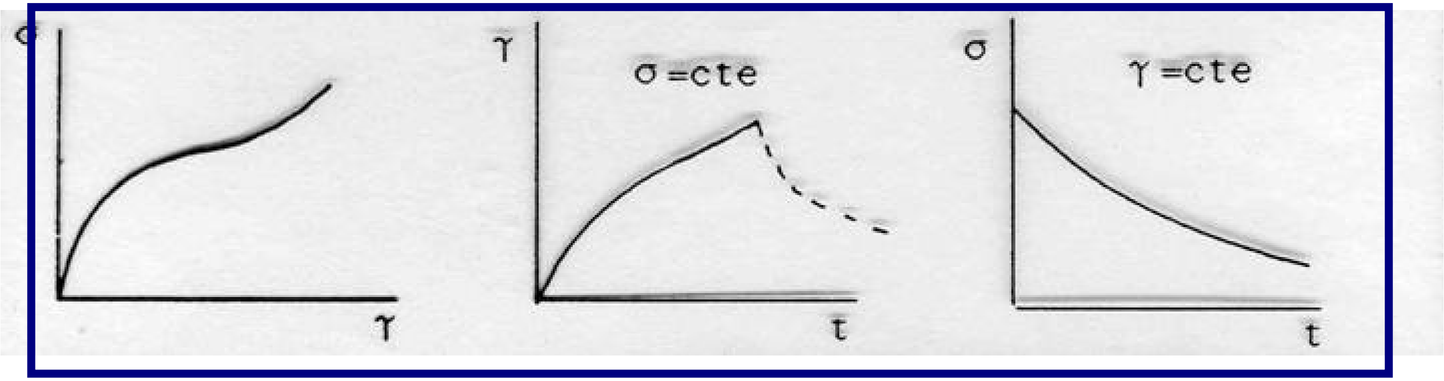
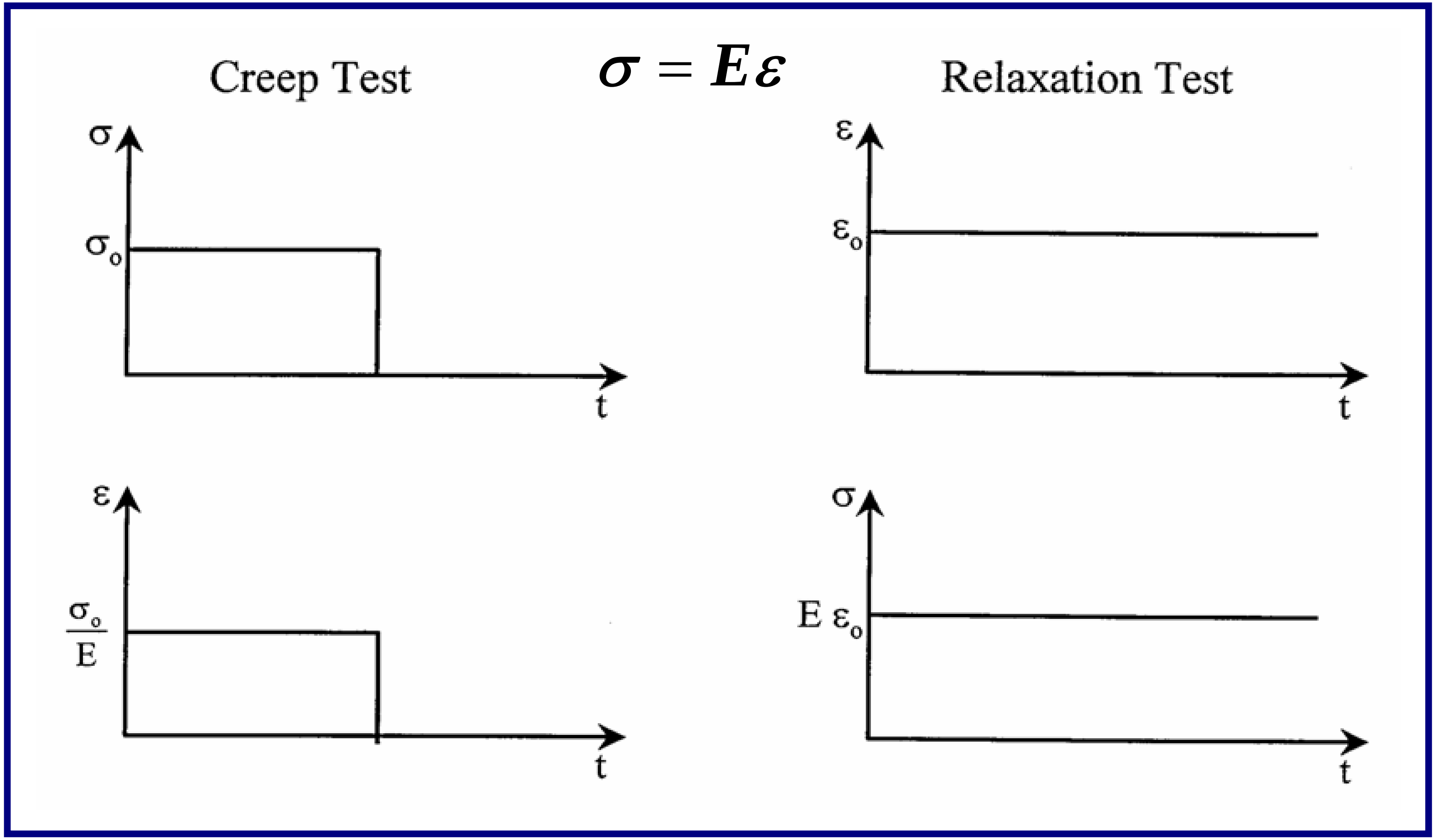
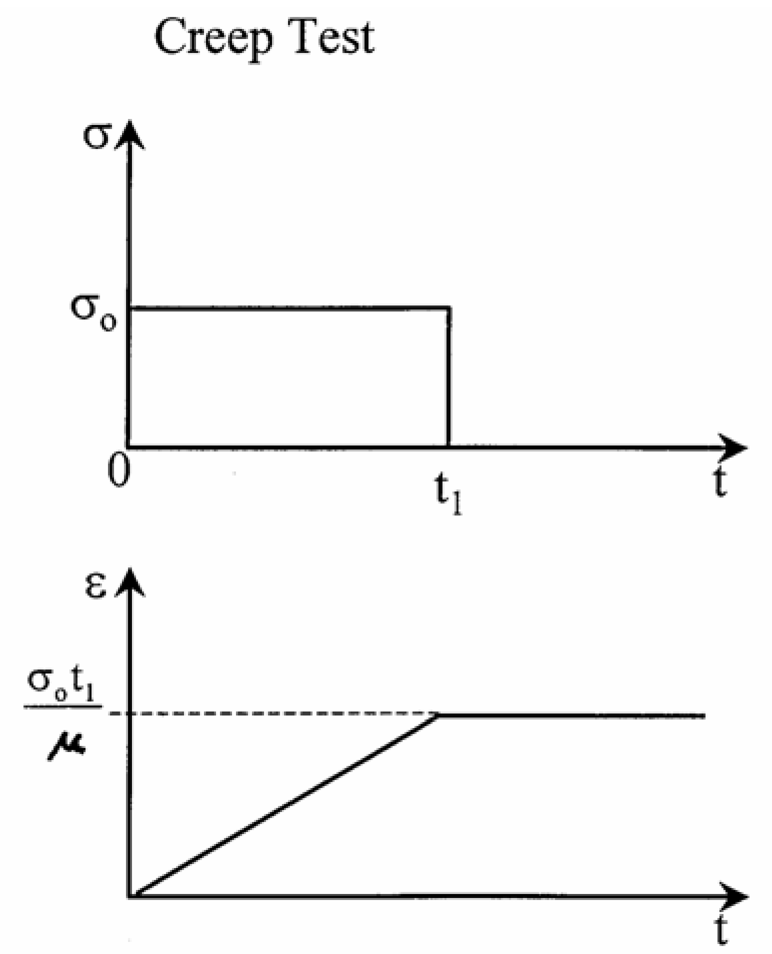
mechanical models
elastic = spring (Hookeian)
viscous = dashpot (Newtonian)
viscoelastic = combination

creep test
response to constant load
relaxation test
response to constant deformation




Creep Test
$\sigma = E\varepsilon$
Relaxation Test
σ
σo
t
ε
εo
t
ε
σo / E
t
σ
E εo
t



$$\sigma = \dot{\varepsilon}\mu$$

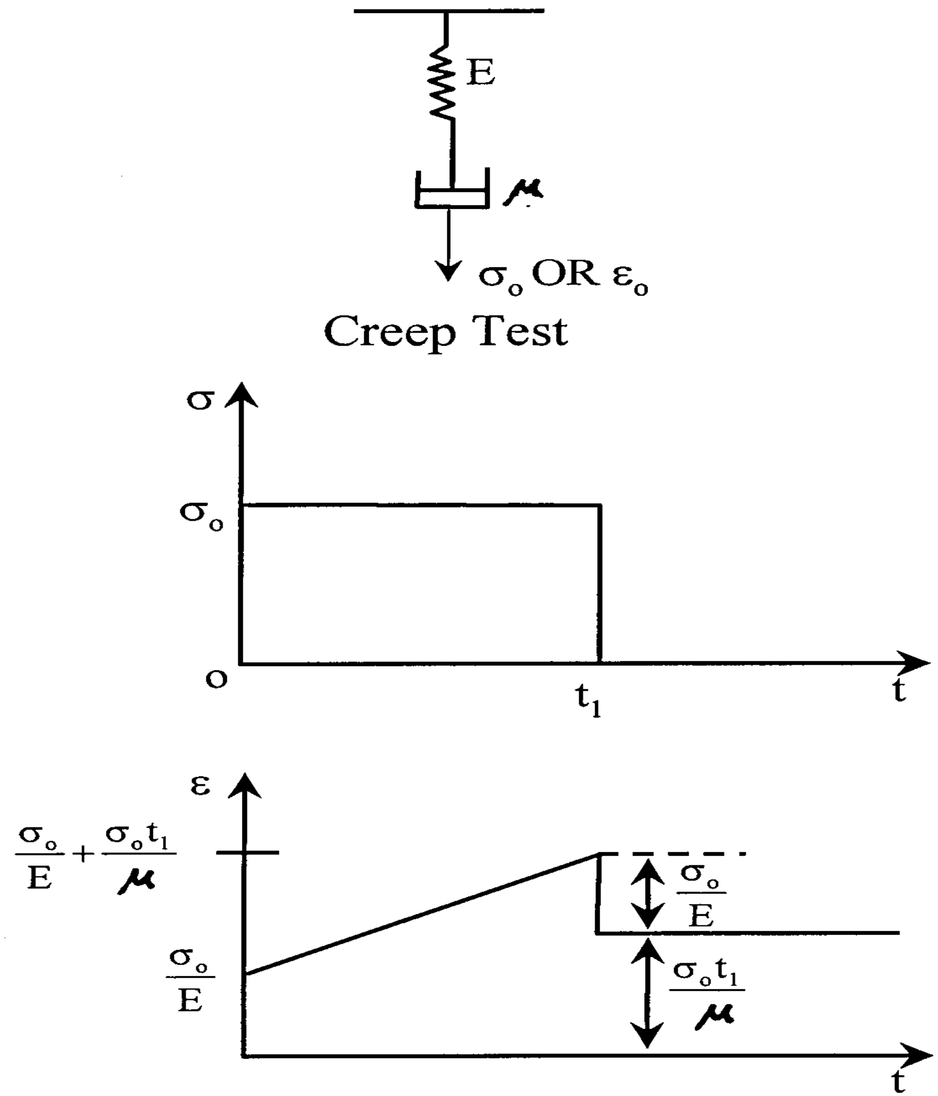
$$\tau = \dot{\gamma}\eta$$

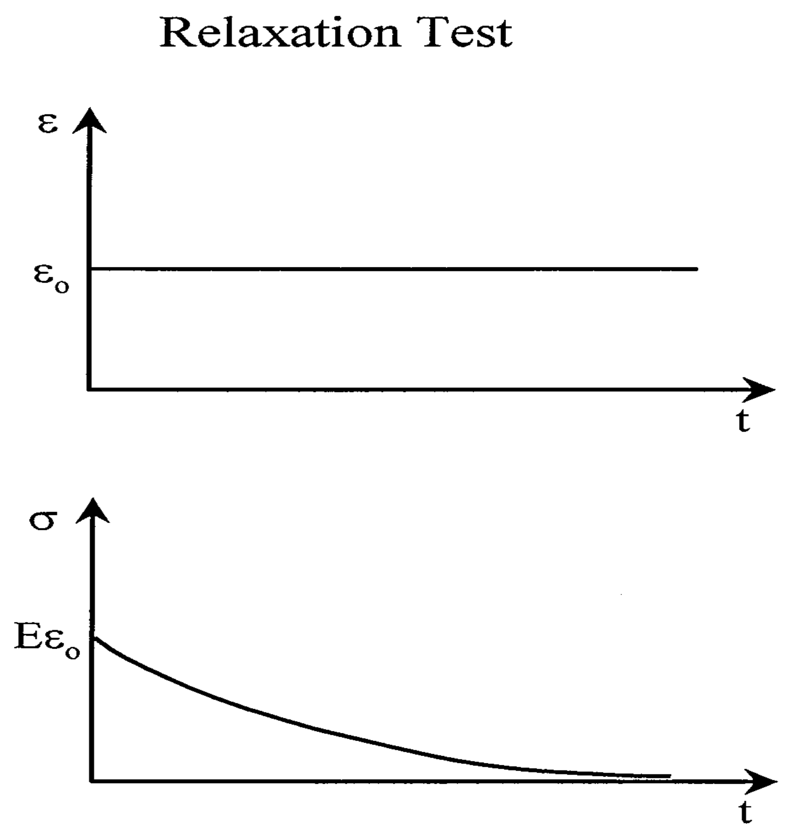
$$\boldsymbol{E = 2(1+\upsilon)G}$$

$$\textbf{for } \boldsymbol{\upsilon = 0.5;\ \mu = 3\eta}$$

Viscoelastic solids (MAXWELL Model)

Elements in Series
Same Stress Acts on Both
Elements





spring + dashpot in series
creep test

$$\varepsilon(t) = \sigma_0(\frac{1}{E} + \frac{t}{\mu})$$

σ_o

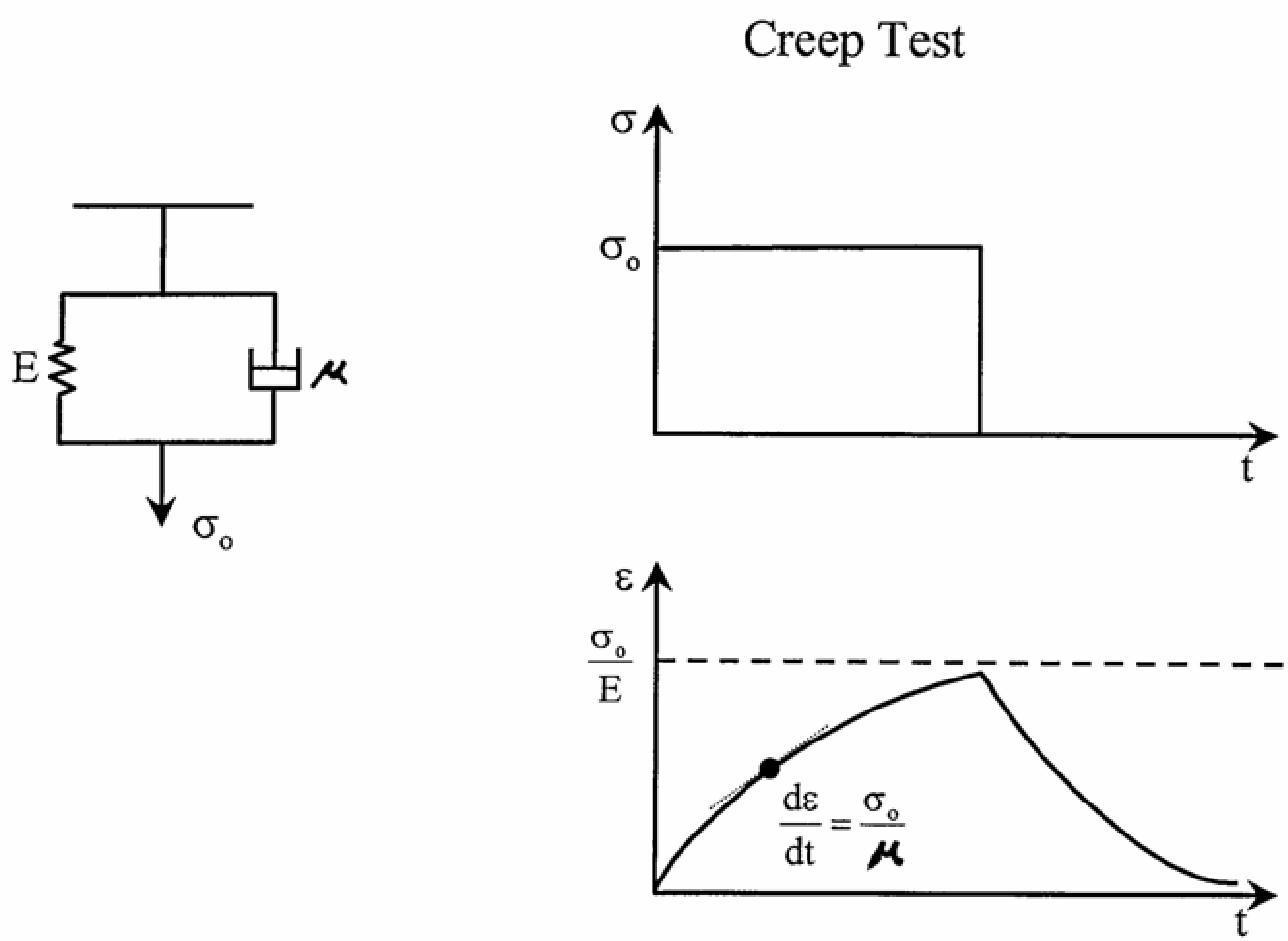
relaxation test

$$\sigma(t) = E\varepsilon_0 e^{-\frac{E}{\mu}t}$$

$$\boxed{\frac{\mu}{E} = \text{relaxation time} = \tau \text{ or } \theta}$$

$\sigma=\sigma_o e\text{-}t/\theta_1$ $\theta_1=\eta_1/E_1$ $\varepsilon=(\sigma_o/E_1)+(\sigma_o/\eta_1)t$ $\sigma=0.37\sigma_o$ per $t=\theta_1$

spring + dashpot in parallel



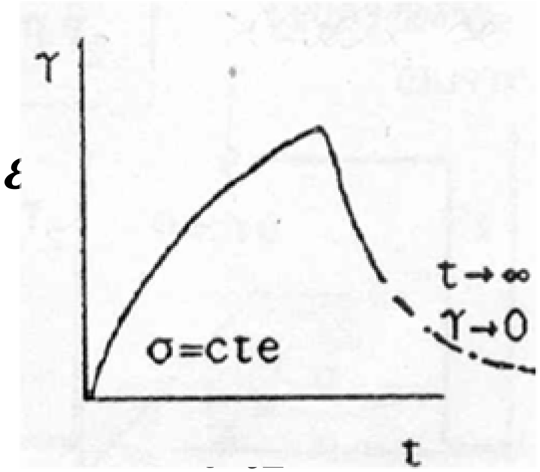
- spring + dashpot in parallel
- creep test

$$\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-\frac{E}{\mu}t}\right)$$

- no relaxation test

$$\frac{\mu}{E} = \text{retardation time} = \tau \text{ or } \theta$$

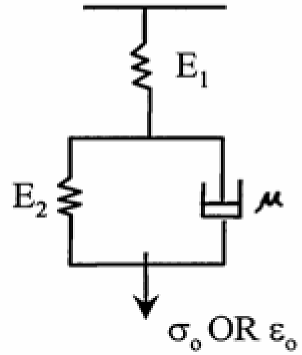
0.63 ε

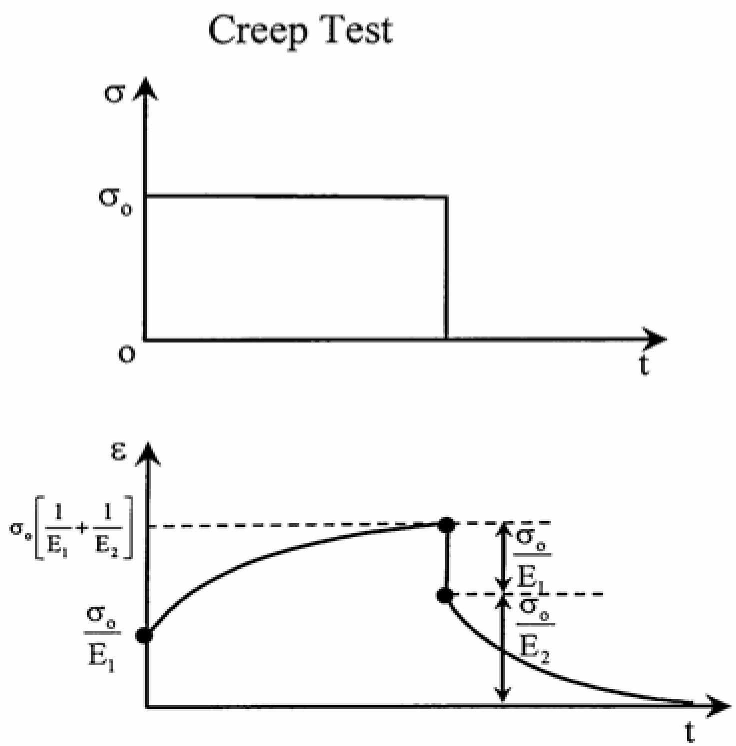


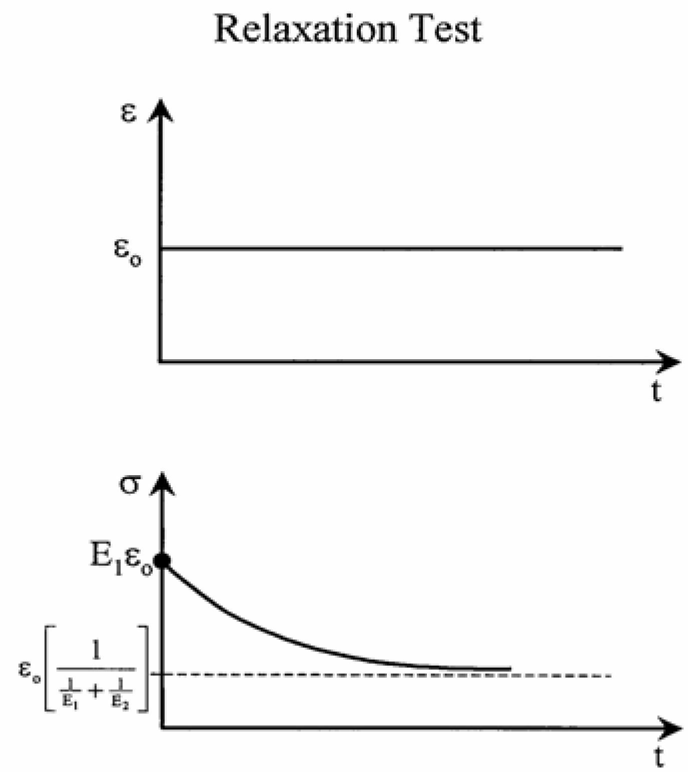
ε=0.63 ε_o per $t=\theta_2$

Viscoelastic solids (STANDARD Model)

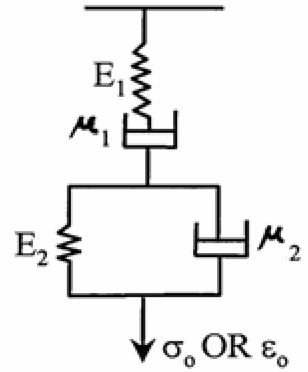
spring + dashpot in parallel
+ spring series





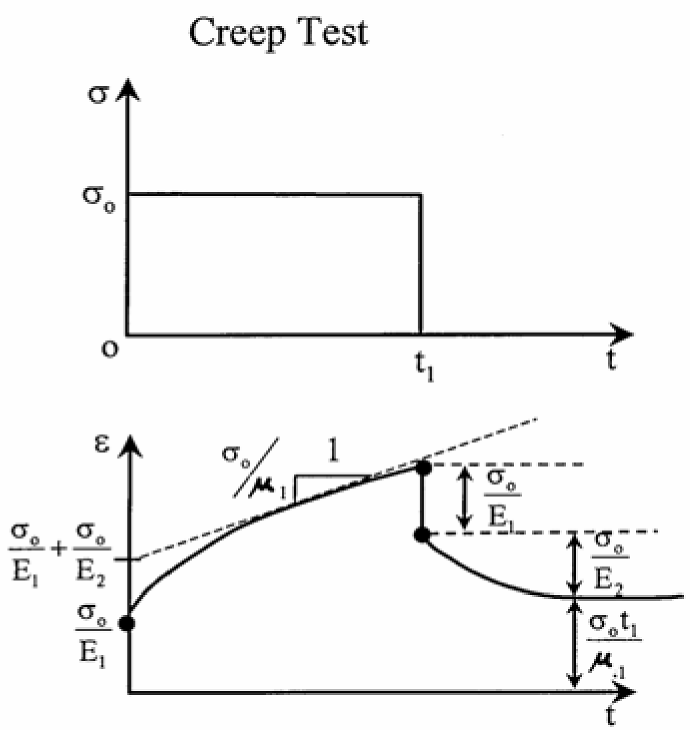


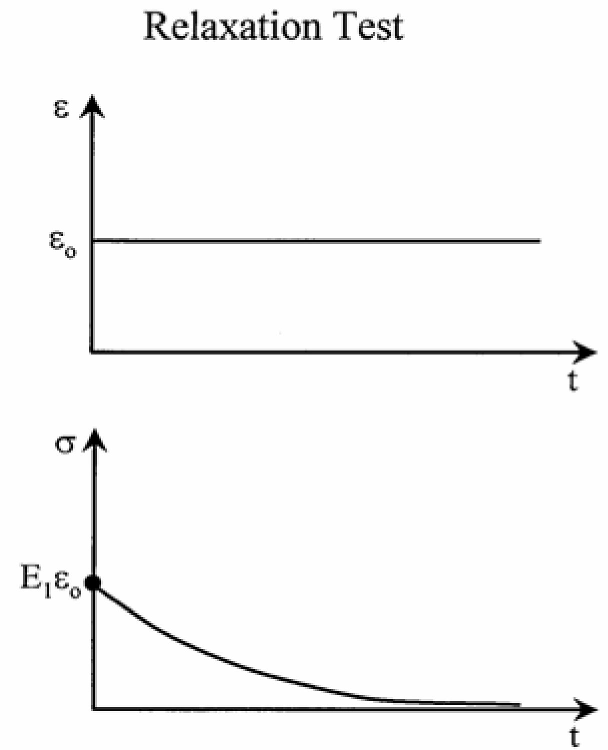
Viscoelastic solids (BURGERS Model)



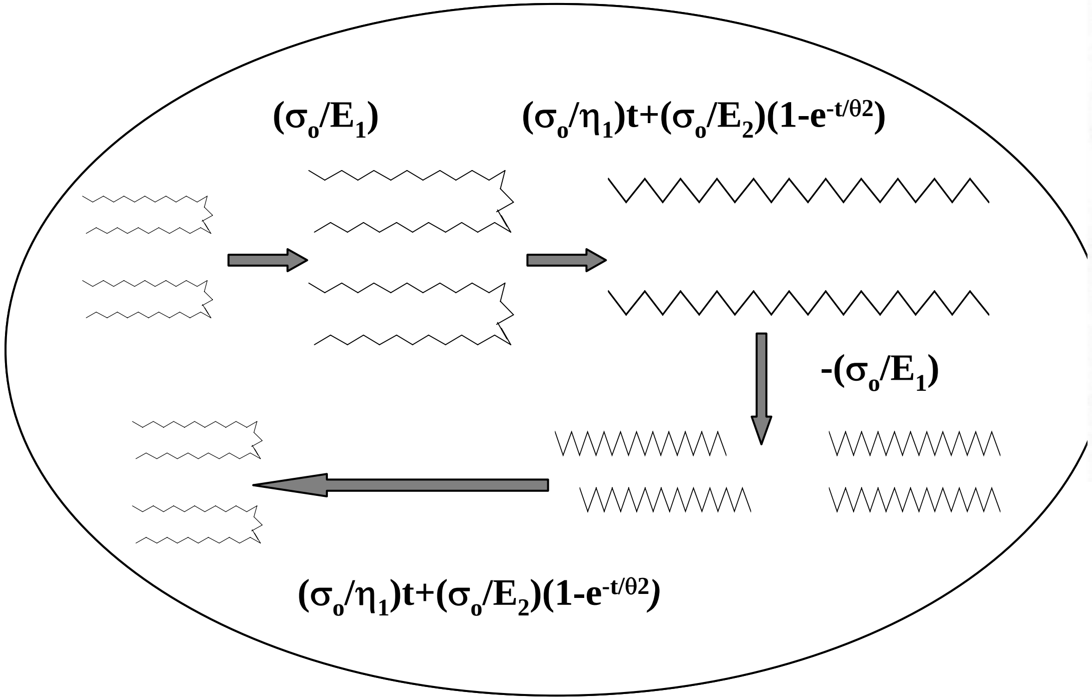
$$\sigma = \sigma o(e\text{-}t/\theta 1+e\text{-}t/\theta 2)$$

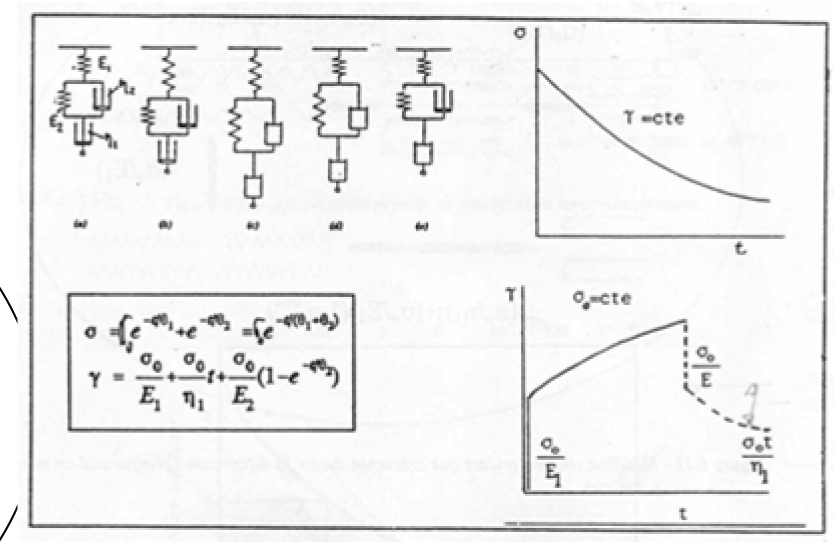
$$\varepsilon = (\sigma o/E1)+(\sigma o/\eta 1)t+(\sigma o/E2)(1-e\text{-}t/\theta 2)$$





Viscoelastic solids (BURGERS Model)





$$\sigma = \sigma o(e\text{-}t/\theta 1+e\text{-}t/\theta 2)$$

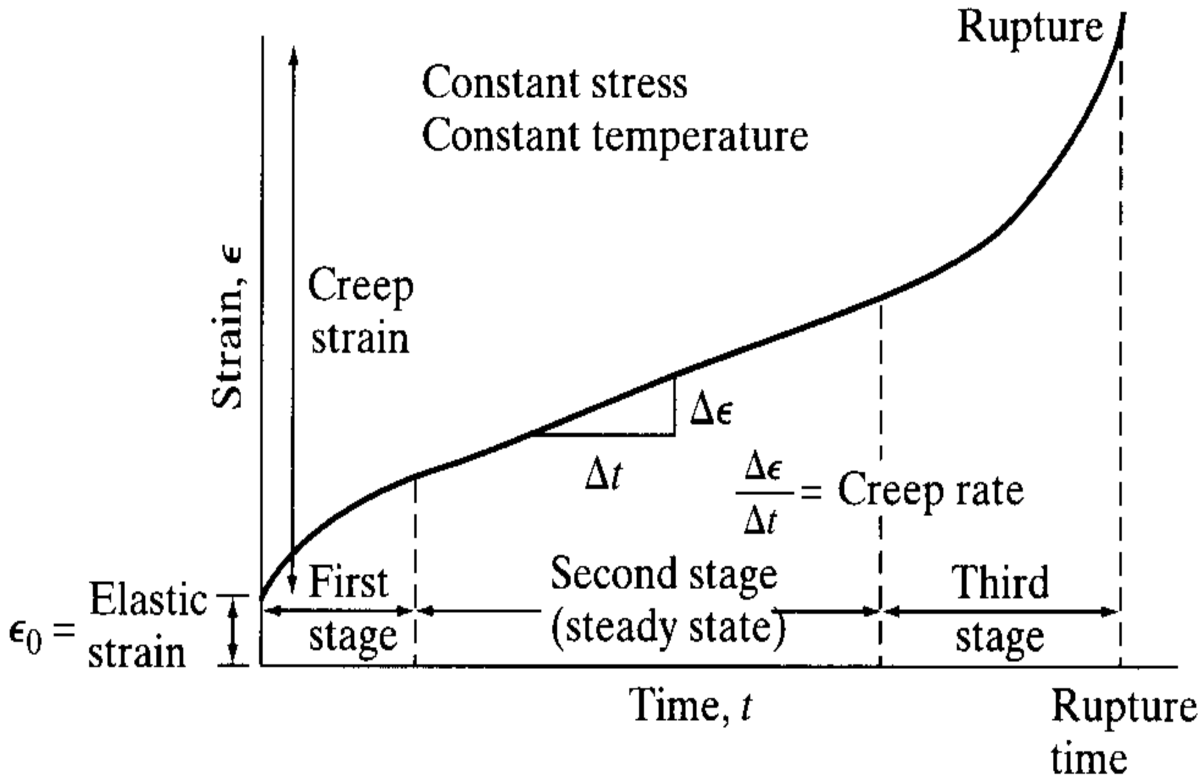
$$\varepsilon= (\sigma o/E1)+(\sigma o/\eta 1)t+(\sigma o/E2)(1\text{-}e\text{-}t/\theta 2)$$

3 Stages f (stress level, temperature)

Stage 1: transient / primary

Stage 2: steady state / secondary

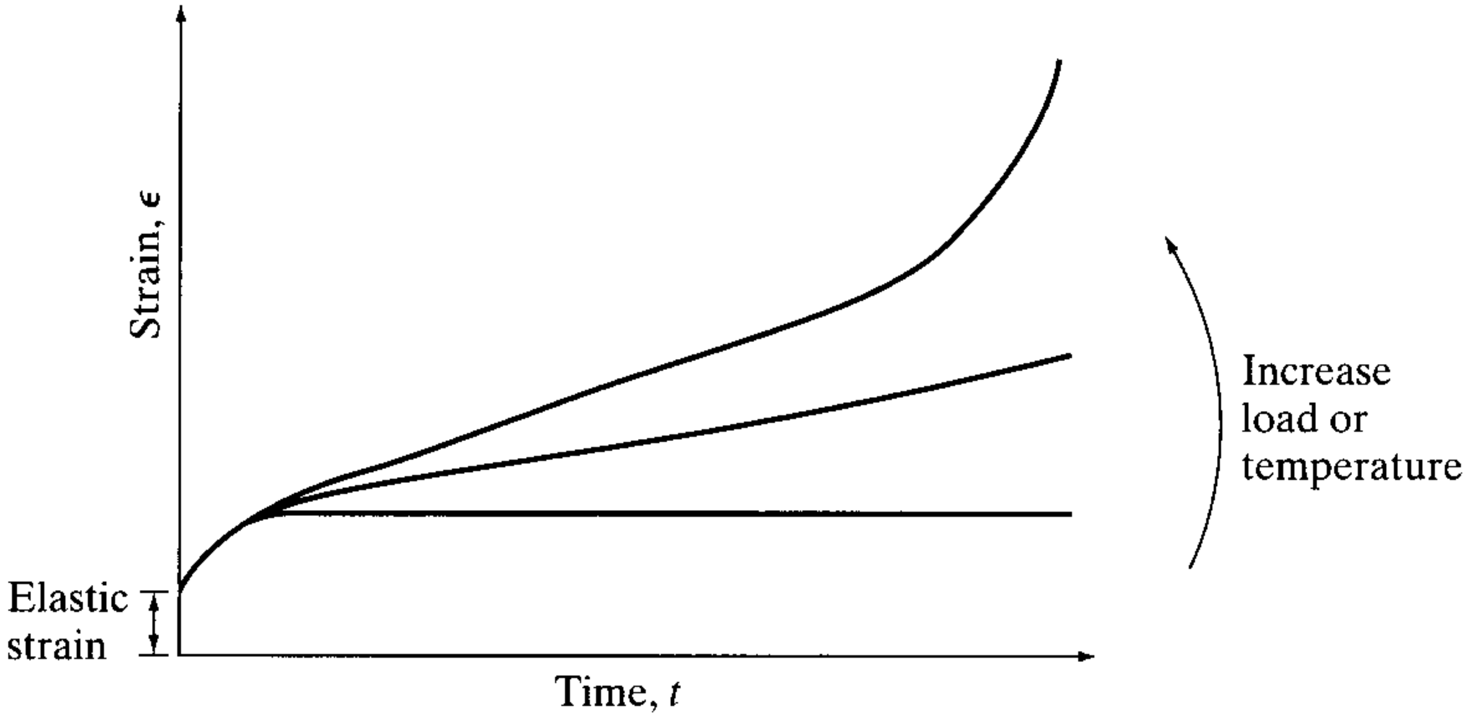
Stage 3: tertiary



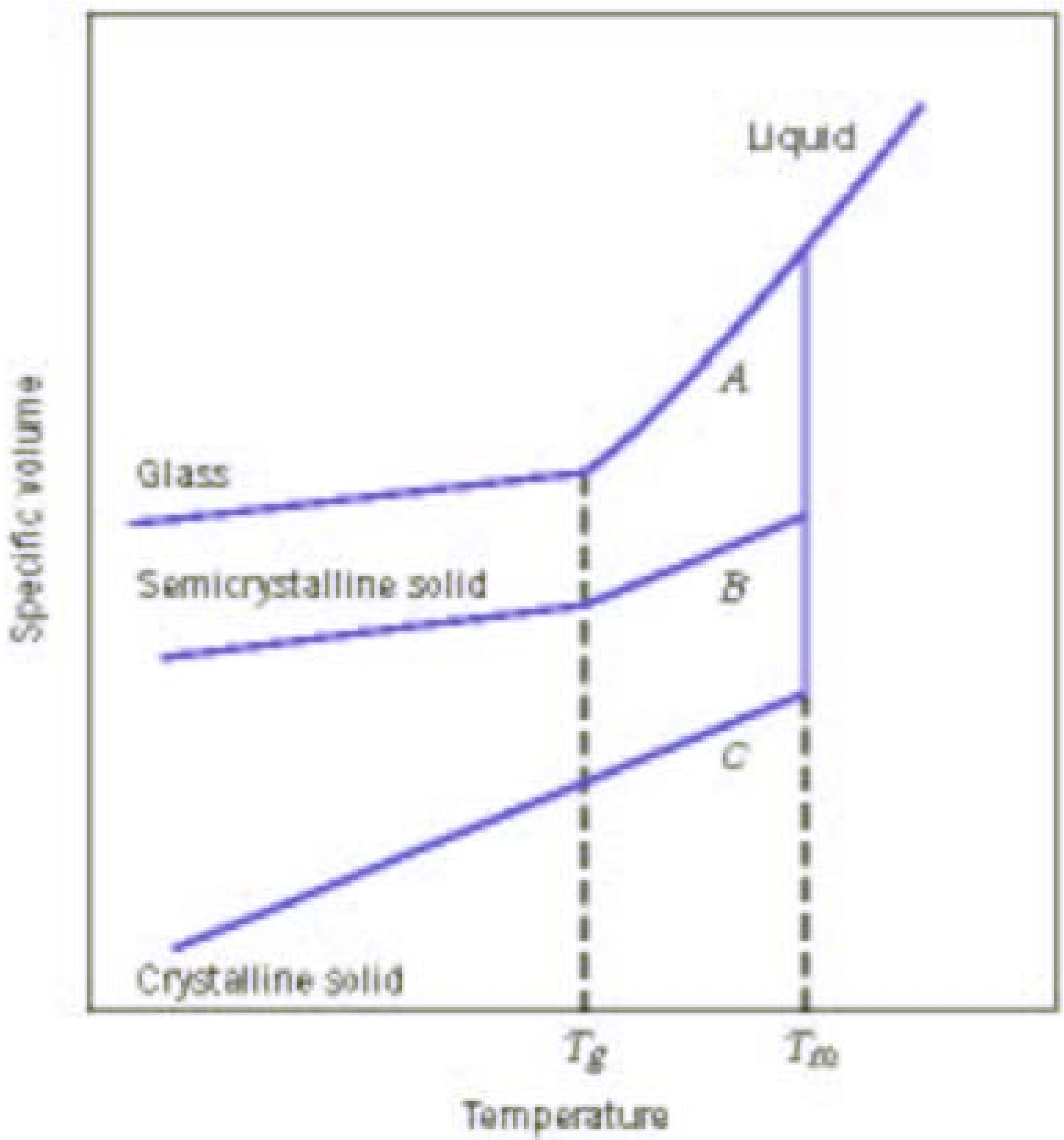
strength and transition temperature

Tmelting (metals), Tglass (polymers)

important during service life if material temperature or stress levels exceed 1/3 of transition temp & strength



CREEP (metalic and polymeric matrices)



CREEP (polymer)

sliding of macromolecules; slow extension of individual chains
thermally activated (low Ea, low Tg)
increased probability of movement
increase temp

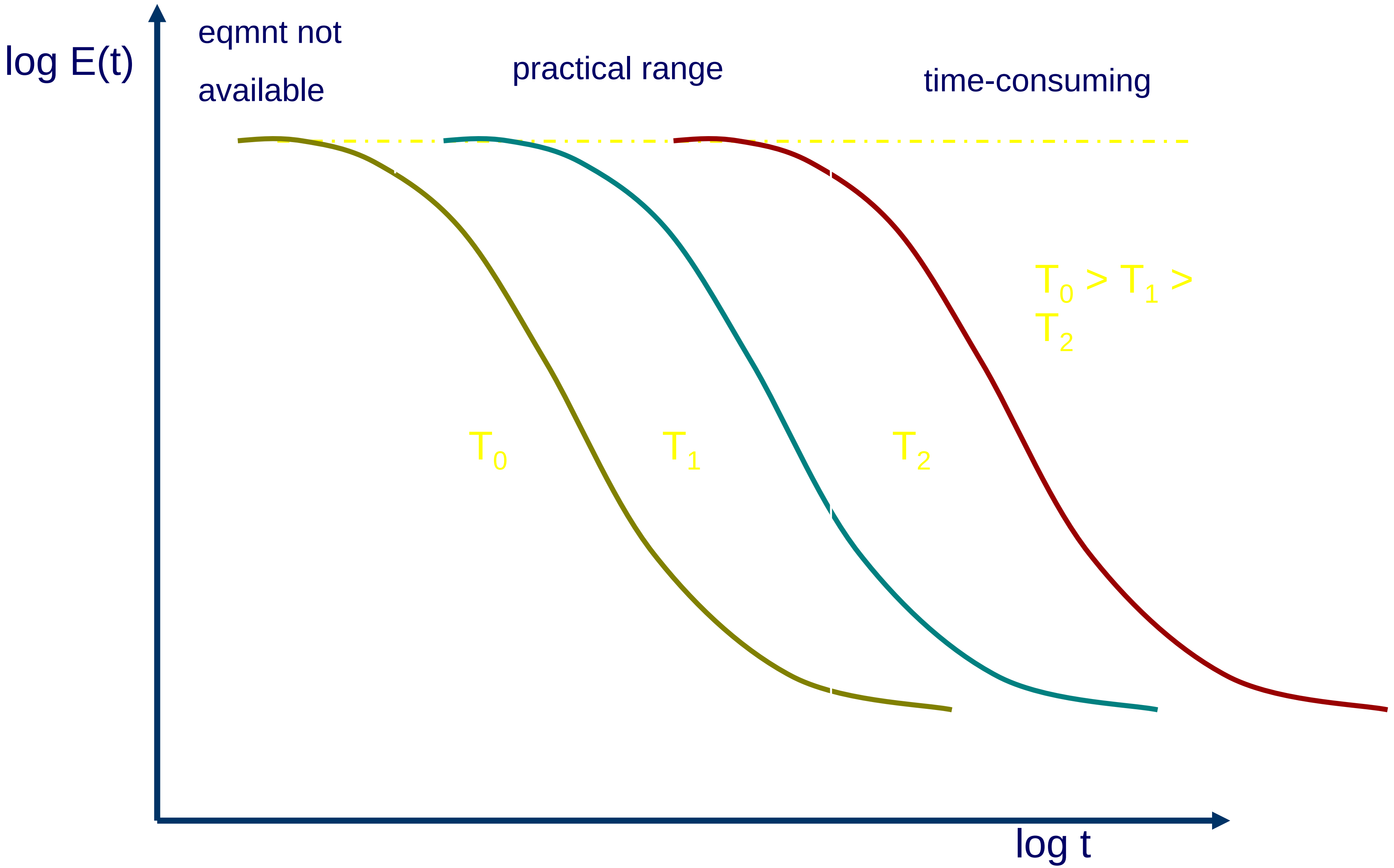
increase time of loading
time-temperature superposition
master reference curve - define properties for long & short times of loading not practical or feasible in laboratory testing

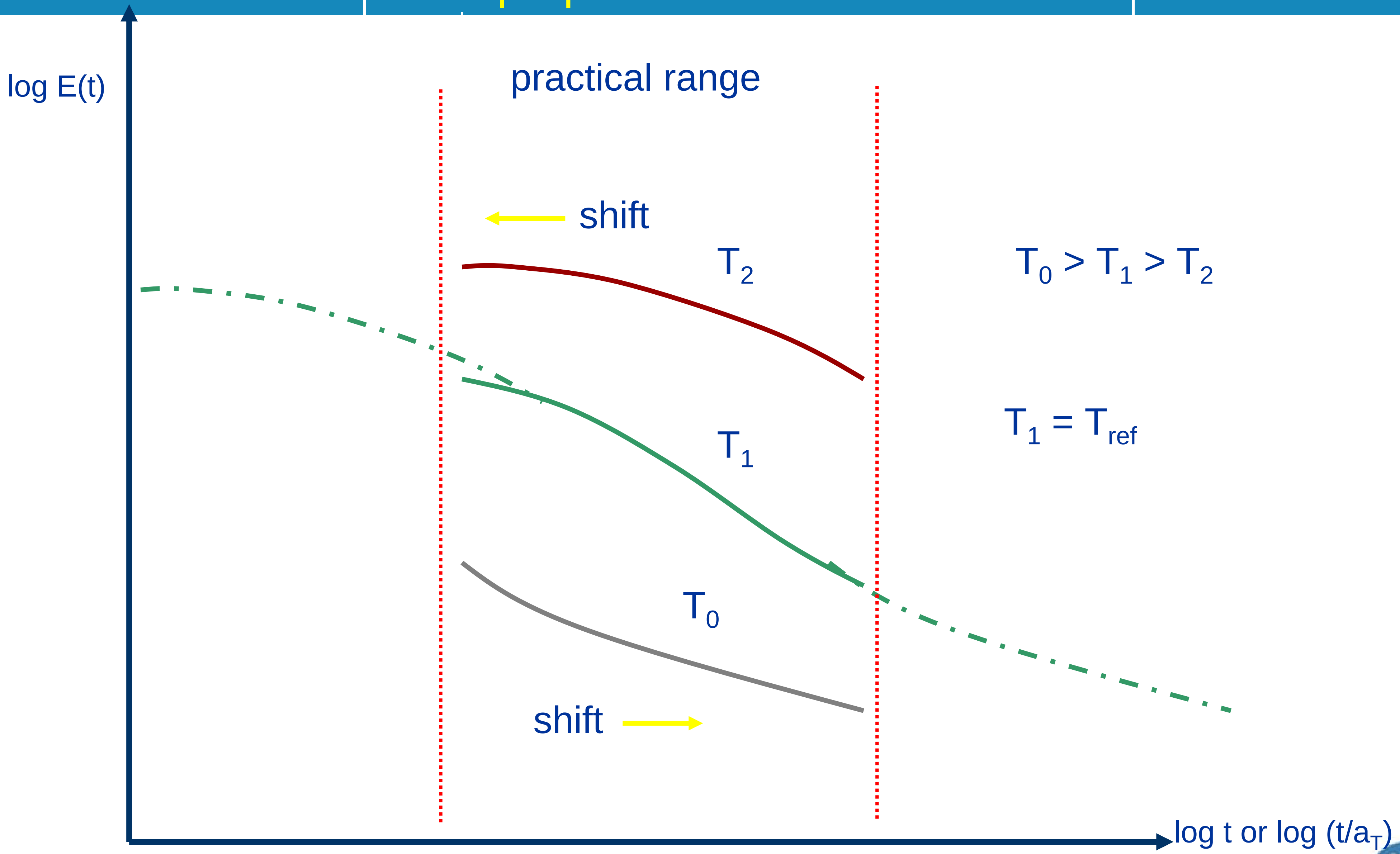
low temp / fast loading = elastic (spring)
high temp / slow loading = viscous (dashpot)
intermediate temp = viscoelastic

stiffness relaxation modulus	$E_r(t,T) = \dfrac{\sigma(t)}{\varepsilon_0}$

creep modulus creep compliance	$E_c(t,T) = \dfrac{\sigma_0}{\varepsilon(t)}$

THERMO-RHEOLOGICALLY SIMPLE




log E(t)
practical range
shift
T_2
$T_0 > T_1 > T_2$
$T_1 = T_{ref}$
T_1
T_0
shift
log t or log (t/a_T)

EXEMPLE 1

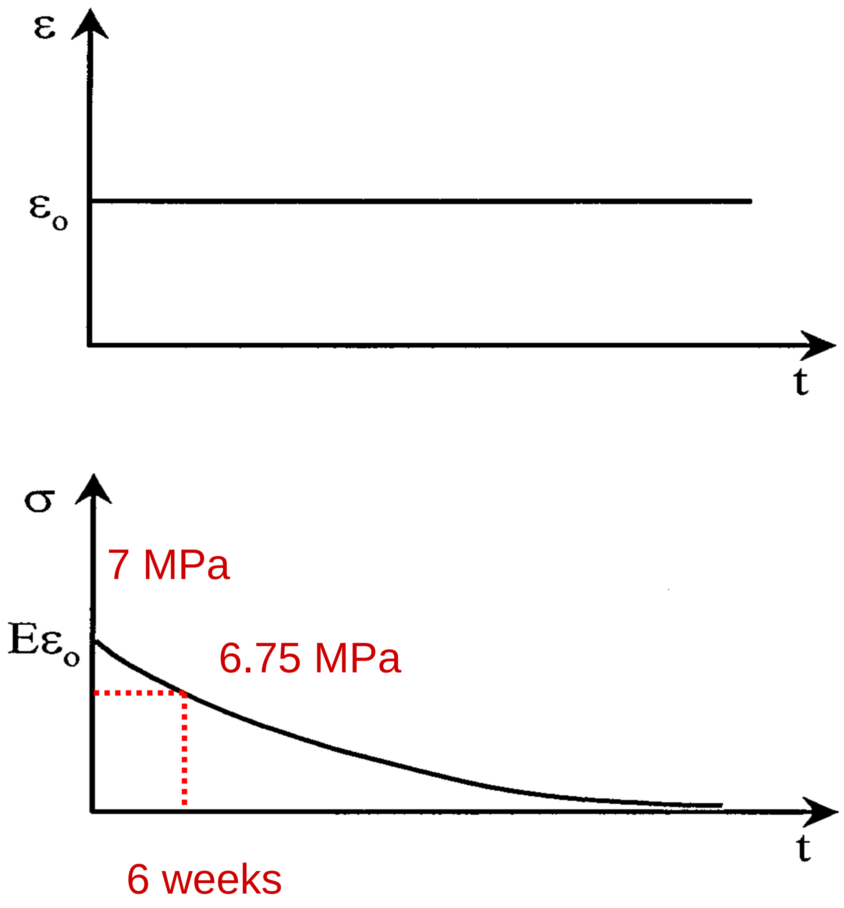
A band is used to hold together a bundle of steel rods for up to a period of one year. If the stress in the band is less than 10 MPa, the rods will not be held together well enough.

Lab tests show that an initial stress of 7 M Pa decreased to 6.75 M Pa in 6 weeks.

Find the minimum initial stress that must be applied.

Which model should be used?

Relaxation Test



Maxwell model

$$\sigma = \sigma_0 e^{-t/\tau}$$

$$6.75 = 7e^{-6/\tau}$$

$$-\frac{6}{\tau} = \ln\left(\frac{6.75}{7}\right) = -0.036; \tau = 165 weeks$$

$$10 = \sigma_0 e^{-52/165} = 0.730\sigma_0$$

$$\sigma_0 = \frac{10}{0.73} = 13.7 MPa$$

EXEMPLE 2

A composite material with a viscoelastic behaviour (Burgers model) with a E1= 10 MPa and a $\mu 1$= 10 MPa·sec. Submitted at a load of 5 MPa, can reach a maximum elongation of 140%. Once it leaves to apply this load the residual elongation will reach the 40%.

What's the elongation at the retardation time ($\theta 2$)?

When the composite will reach this residual elongation?

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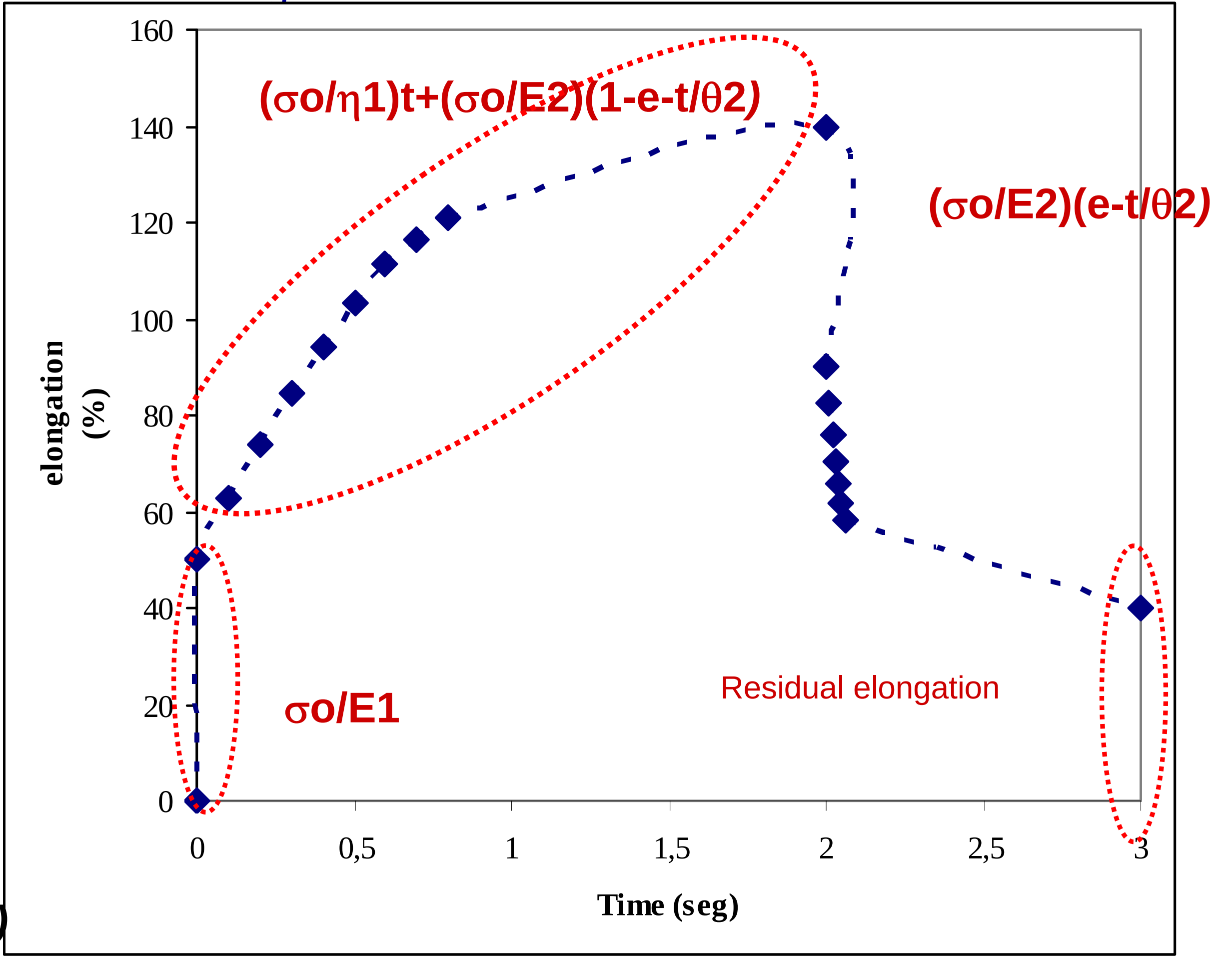
When the composite will reach this residual elongation?



$\sigma = \sigma o(e\text{-}t/\theta 1 + e\text{-}t/\theta 2)$

$\varepsilon = (\sigma o/E1) + (\sigma o/\eta 1)t + (\sigma o/E2)(1 - e\text{-}t/\theta 2)$

EXEMPLE 2

What's the elongation at the retardation time ($\theta 2$)? $\theta 2 = \mu 2/E2$

$\varepsilon = (\sigma o/E1)+(\sigma o/\eta 1)t+(\sigma o/E2)(1-e-t/\theta 2)$

ε inst= $\sigma o/E1$ =0.5 (50%)

ε residual= $\sigma o/\eta 1)t$ = 0.4 (40%)

If the maximum elongation is 140% middel elongation

will be ε m =($\sigma o/E2)(1-e-t/\theta 2)$=*0.5 (50%)*