ΠΑΡΑΔΕΙΓΜΑ 4ο

Να βρεθεί ο αντίστροφος μετασχηματισμός Laplace της συνάρτησης

$$F(s) = \frac{3s+1}{(s-1)(s^2+1)}$$

ΛΥΣΗ (1ος ΤΡΟΠΟΣ).

$$s^2+1=0 \Rightarrow s^2=-1 \Rightarrow \sqrt{s^2}=\pm\sqrt{-1} \Rightarrow s_{1,2}=\pm j$$

Επομένως η

$$F(s)=\frac{3s+1}{(s-1)(s^2+1)}=\frac{3s+1}{(s-1)(s-j)(s+j)}\equiv\frac{K_1}{s-1}+\frac{K_2}{s-j}+\frac{K_3=\overline{K_2}}{s+j}$$

$$K_1=\left[\cancel{(s-1)}\frac{(3s+1)}{\cancel{(s-1)}(s^2+1)}\right]_{s=1}=\frac{(3)(1)+1}{(1)^2+1}=\frac{4}{2}=2.$$

$$K_2=\left[\cancel{(s-j)}\frac{(3s+1)}{(s-1)\cancel{(s-j)}(s+j)}\right]_{s=+j}=\frac{3j+1}{(j-1)(2j)}=\frac{3j+1}{2j^2-2j}=$$

$$=\frac{1+3j}{2(-1)-2j}=\frac{1+3j}{-2-2j}=\frac{(1+3j)(-2+2j)}{(-2-2j)(-2+2j)}=\frac{-2+2j-6j+6j^2}{(-2)^2-(+2j)^2}=$$

$$=\frac{-2+2j-6j+6(-1)}{4-(4j^2)}=\frac{-2+2j-6j-6}{4-[4(-1)]}=\frac{-8-4j}{4+4}=$$

$$=\frac{-8-4j}{8}=-1-\frac{1}{2}j=-1-0{,}5j$$

Επομένως $K_3=\overline{K_2}=-1+0{,}5j$

Συνεπώς η

$$F(s) = \frac{2}{s-1} + \frac{(-1-0{,}5j)}{s-j} + \frac{(-1+0{,}5j)}{s+j} \Rightarrow$$

$$\Rightarrow F(s) = 2\cdot\frac{1}{s-1} - (1+0{,}5j)\cdot\frac{1}{s-j} + (-1+0{,}5j)\cdot\frac{1}{s+j} \Rightarrow$$

$$\Rightarrow L^{-1}F(s) = L^{-1}\left\{2\cdot\frac{1}{s-1}\right\} + L^{-1}\left\{-(1+0{,}5j)\frac{1}{s-j}\right\} + L^{-1}\left\{(-1+0{,}5j)\cdot\frac{1}{s+j}\right\} \Rightarrow$$

$$\Rightarrow L^{-1}F(s) = 2L^{-1}\left\{\frac{1}{s-1}\right\} - (1+0{,}5j)L^{-1}\left\{\frac{1}{s-j}\right\} + (-1+0{,}5j)L^{-1}\left\{\frac{1}{s+j}\right\} \Rightarrow$$

$$\Rightarrow f(t) = 2e^{-(-t)} - (1+0{,}5j)e^{-(-j)t} + (-1+0{,}5j)e^{-jt} \Rightarrow$$

$$\Rightarrow f(t) = 2e^{t} - e^{jt} - 0{,}5je^{jt} - e^{-jt} + 0{,}5je^{-jt} \Rightarrow$$

$$\Rightarrow f(t) = 2e^{t} - (e^{jt} + e^{-jt}) - 0{,}5j(e^{jt} - e^{-jt}) \Rightarrow$$

$$\Rightarrow f(t) = 2e^{t} - 2\cos t - 0{,}5j\,2j\sin t \Rightarrow$$

$$\Rightarrow \boxed{f(t) = 2e^{t} - 2\cos t + \sin t}$$

ΛΥΣΗ ($2^{ος}$ ΤΡΟΠΟΣ).

$$F(s)=\frac{3s+1}{(s-1)(s^2+1)} \equiv \frac{K_1}{s-1}+\frac{K_2 s+K_3}{s^2+1}$$

$$K_1=\left[\cancel{(s-1)}\,\frac{(3s+1)}{\cancel{(s-1)}(s^2+1)}\right]_{s=1}=\frac{3+1}{1+1}=\frac{4}{2}=2$$

Αντικαθιστώ $K_1=2$ στην $F(s)$, άρα

$$F(s)=\frac{\overbrace{3s+1}^{1}}{(s-1)(s^2+1)} \equiv \frac{\overbrace{2}^{s^2+1}}{s-1}+\frac{\overbrace{K_2 s+K_3}^{s-1}}{s^2+1} \Rightarrow$$

$$\Rightarrow\ 3s+1 \equiv 2(s^2+1)+(s-1)(K_2 s+K_3) \Rightarrow$$

$$\Rightarrow\ 3s+1 \equiv 2s^2+2+K_2 s^2+K_3 s-K_2 s-K_3 \Rightarrow$$

$$\Rightarrow\ 3s+1 \equiv (K_2+2)s^2+(K_3-K_2)s+(2-K_3)$$

Εξισώνω τους συντελεστές των ομοίων όρων.

$K_2+2=0 \quad (1) \Longrightarrow \boxed{K_2=-2}$

$K_3-K_2=3 \quad (2) \longrightarrow$ ΕΠΑΛΗΘΕΥΣΗ $1-(-2)=3$

$2-K_3=1 \quad (3) \Longrightarrow -K_3=1-2 \Rightarrow -K_3=-1 \Rightarrow \boxed{K_3=1}$

Επομένως η

$$F(s)=\frac{2}{s-1}+\frac{(-2s+1)}{s^2+1} \Longrightarrow$$

$$\Rightarrow F(s) = 2\cdot\frac{1}{s-1} - 2\cdot\frac{s}{s^2+1} + \frac{1}{s^2+1}$$

$$L^{-1}\{F(s)\} = L^{-1}\left\{2\cdot\frac{1}{s-1}\right\} + L^{-1}\left\{(-2)\cdot\frac{s}{s^2+1}\right\} + L^{-1}\left\{\frac{1}{s^2+1}\right\} \Rightarrow$$

$$\Rightarrow L^{-1}\{F(s)\} = 2L^{-1}\left\{\frac{1}{s-1}\right\} - 2L^{-1}\left\{\frac{s}{s^2+1}\right\} + L^{-1}\left\{\frac{1}{s^2+1}\right\} \Rightarrow$$

$$\Rightarrow f(t) = 2e^{-(-t)} - 2\cos t + \sin t \Rightarrow$$

$$\Rightarrow \boxed{f(t) = 2e^{t} - 2\cos t + \sin t}$$